

Fermion Mixing and Mass Hierarchy as Consequences of Mass Matrix Rotation

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Abstract

It is shown that a fermion mass matrix changing in orientation (rotating) with changing scales can give a simple yet near-quantitative explanation for quark mixing, neutrino oscillations and the fermion mass hierarchy.

Like other quantities in quantum field theory such as the familiar running coupling constant, the fermion mass matrix also varies with changing scales. That its eigenvalues, namely the mass values, do actually run has already been verified experimentally in certain cases [1]. It should thus come as no surprise that the fermion mass matrix may also change its orientation in generation space (rotate) as the scale changes.

Indeed, even in the situation when there are no other forces involved than those currently studied in the Standard Model, it is easily seen that the fermion mass matrices must rotate with changing scales once given nontrivial mixing between the U - and D -fermion states. For instance, the U mass matrix satisfies a renormalization group equation [2]:

$$16\pi^2 \frac{dU}{dt} = -\frac{3}{2} D D^\dagger U + \dots \quad (1)$$

which contains a term on the right which is nondiagonal even when U is diagonal if the D mass matrix is related to U by a nontrivial mixing matrix. Hence, as the scale changes, a U matrix diagonal at one scale will no longer remain so at another scale, or in other words, it will “rotate” with changing scales, as claimed. And since nontrivial mixing has long been established for quarks, while for leptons it has been strongly indicated if not already confirmed by recent experiment [3, 4], we conclude that both the quark and lepton mass matrices must rotate with changing scales even in the traditional Standard Model scenario.

Looking further afield, there are good reasons to consider the possibility of having further, perhaps more direct, mechanisms driving the mass matrix rotation. The mere fact that different generations of fermions can rotate into one another, as the mass matrix rotation implies, already means that they are not distinct entities as once conceived but just different manifestations of the same object, like the different colours of a quark, related presumably by some continuous “horizontal” symmetry [5]. And if this symmetry is gauged, as all other known continuous symmetries seem to be, then it would give rise to new forces which can change the generation index and hence contribute to the rotation of the fermion mass matrices. And these contributions would be over and above those driven by nontrivial mixing via (1).

Indeed, what we wish to point out in this note is a, to us, attractive possibility that it is the rotation of the mass matrix (due presumably to some new as yet unfamiliar forces) which is giving rise to fermion mixing rather than the other way round. As we shall show, this can give us an immediate explanation not only for the remarkable fermion mixing pattern observed

in experiment but also for the otherwise puzzling hierarchical fermion mass spectrum.

The reason why one can at all entertain this possibility is that once the mass matrix rotates then it can generate fermion mixing and nonzero masses for the lower generation fermions even when one starts with neither. To see this, one will need first to re-examine some basic premises which have to be revised in view of the mass matrix rotation. When the mass matrix has a scale-independent orientation (i.e. when it does not rotate), the state vectors in generation space representing the different generations are trivially defined as the eigenvectors of the mass matrix. However, if the mass matrix rotates, then so will its eigenvectors, and the above definition of the generation states becomes imprecise, for it will need to be specified at what scale(s) these states are to be taken as the mass eigenvectors. Consider first as example the t quark, which we can take as the eigenstate of the U -type quark mass matrix m^U with the largest eigenvalue, say m_1^U . But this value m_1^U depends on the scale μ , and one usually defines the t quark mass m_t as the value at the scale equal to the value itself, i.e. when $\mu = m_1^U(\mu)$. It seems natural then to define also the t quark state vector as the corresponding eigenvector of m^U also at the same scale. Similarly, one would define the state vector of the b quark as the eigenvector of the D -type quark mass matrix with the largest eigenvalue taken at the scale $\mu = m_1^D(\mu)$. If so, the state vectors for the t and b quarks would be defined at different scales. Hence, even with the additional ansatz that the U and D mass matrices have always exactly the same orientation (i.e. aligned eigenvectors) at the same scale, the state vectors of t and b will be different since the eigenvector would have rotated from the scale at which the t state vector is defined to the scale at which the b state vector is defined. In other words, there will be nontrivial mixing between the t and b states since the mixing (CKM) matrix element defined as $V_{tb} = \langle t|b \rangle$ will differ from unity.¹

¹We have defined the CKM matrix here as the overlap matrix between the triads of physical state vectors of respectively the U and D fermions, i.e. the same as for non-rotating mass matrices. Apart from being conceptually simple, this definition also corresponds to what is usually actually measured in experiment. For example, the element V_{tb} is inferred experimentally in t decay from its branching ratio to b , where b in the final state is identified also by the branching ratios in its decay, which means the state vector of the decaying particle is in each case evaluated at the scale equal to its mass, as it is done here. In the literature, the CKM matrix is sometimes defined instead as the overlap matrix between the eigenvectors of the U and D mass matrices taken all at the same scale, which matrix is then scale-dependent, since the eigenvectors are themselves scale-dependent. This alternative definition is of course perfectly justified if used consis-

Similarly, one sees that when the mass matrix rotates, lower fermion generations will acquire masses even when they start with none. To illustrate the point, it is sufficient to consider only two generations, i.e., say, for the U quark case, only the t and c quarks. We have already defined the t state vector above at the scale of the t mass, and since there are only 2 states, it follows that the c state vector is also uniquely defined as the vector orthogonal to $|t\rangle$. This is necessary since c is quantum mechanically an independent state from t . Suppose now that the U quark mass matrix at any scale has only one nonzero eigenvalue, which at the scale $\mu = m_t$ we take to be m_t . It follows therefore by assumption that the c state taken at the same scale will have zero mass. But this should not be taken as the mass of the c quark, for by the definition above, the physical mass of any fermion state is to be defined at the scale of the mass itself, namely for the c state it should be taken at the scale $\mu = m_c$, not at the scale $\mu = m_t$. At $\mu = m_c$, however, the sole eigenvector $|v_1^U\rangle$ of the mass matrix m^U with nonzero eigenvalue m_1^U would have already rotated to a different direction, as depicted in Figure 1, and acquired a component in the $|c\rangle$ direction giving thus a nonzero mass value $\langle c|m^U|c\rangle$ taken at this scale, as anticipated. A similar mechanism for generating lower generations masses would hold of course for other fermion types, which we shall henceforth refer to as the “leakage” mechanism.

Given that both nontrivial mixing and nonzero lower generation masses can result from a rotating mass matrix, and both of these effects are generally small and are in any case otherwise unexplained, it makes sense to enquire whether they can in fact all be obtained in this way. One can approach the problem empirically since, as we shall show, many of the relevant quantities have already been measured and need only to be arranged and interpreted in a manner appropriate for the present purpose. This will be done first in a simplified situation with only 2 generations, namely the 2 heaviest, in each fermion-type, which simplification will be shown later to approximate already very well the actual 3-generation situation. This makes the analysis much more transparent since the problem then becomes planar and there is only one rotation angle and no phases to consider [6, 7]. We have then the pictures shown in Figures 2 and 1 for obtaining respectively mixing matrix elements and lower generation masses.

tently and is unambiguously related to the experimentally measured mixing matrix if it is known how the mass matrices rotate. If the rotation is slow in the energy range of fermion masses, then the two definitions are nearly equal, but if the rotation is of the order considered here, the difference is important, in which case the definition adopted in this paper seems to us more convenient.

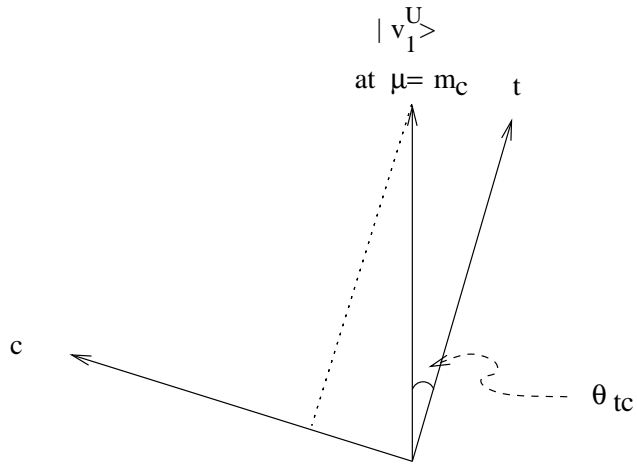


Figure 1: Obtaining lower generation masses by the “leakage” mechanism

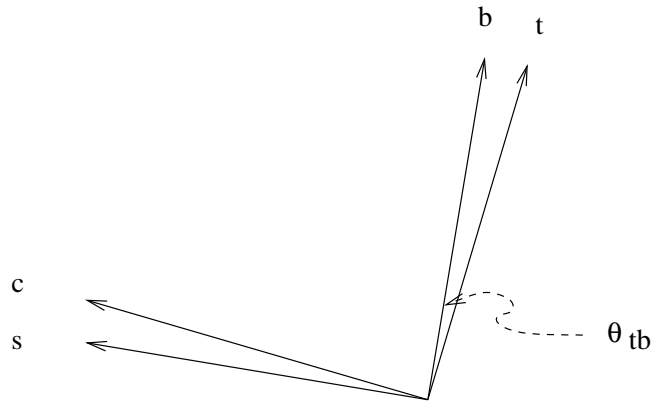


Figure 2: Obtaining mixing matrices from mass matrix rotation

Consider first mixing matrix elements. Suppose from the scale of the t mass to that of the b mass, the mass matrix has rotated by an angle θ_{tb} . We have then the dyads of state vectors shown in Figure 2 for respectively the U - and D -type quarks. One easily obtains then the CKM elements as: $V_{tb} = \cos \theta_{tb}$ and $|V_{ts}| = |V_{cb}| = \sin \theta_{tb}$. From the measured values of these elements given in the latest databook [8], namely:

$$|V_{tb}| = 0.9990 - 0.9993, \quad |V_{ts}| = 0.035 - 0.043, \quad |V_{cb}| = 0.037 - 0.043, \quad (2)$$

one gets thus from each an estimate of the rotation angle, respectively:

$$\theta_{tb} = 0.0374 - 0.0447, \quad 0.0350 - 0.0430, \quad 0.0370 - 0.0430, \quad (3)$$

the values obtained being fully consistent with one another. (One notes that from the same Figure 2, one could deduce in principle also $V_{cs} = \cos \theta_{tb}$, but this will be seen, in contrast to the 3 other mixing elements already considered, to be a poor approximation receiving large nonplanar corrections when all 3 generations are taken into account.)

Consider next the second generation masses obtained by the leakage mechanism. Suppose from the scale of the t mass to that of the c mass, the mass matrix has rotated by an angle θ_{tc} , then one sees from Figure 1 that $m_c/m_t = \sin^2 \theta_{tc}$. Hence, from the measured values of m_t and m_c given in [8], namely:

$$m_t = 174.3 \pm 5.1 \text{ GeV}, \quad m_c = 1.15 - 1.35 \text{ GeV}, \quad (4)$$

one obtains the estimate:

$$\theta_{tc} = 0.0801 - 0.0894. \quad (5)$$

Similarly, from the measured values from [8]:

$$m_b = 4.0 - 4.4 \text{ GeV}, \quad m_s = 75 - 170 \text{ MeV}, \quad (6)$$

one obtains the estimate:

$$\theta_{bs} = 0.1309 - 0.2076, \quad (7)$$

the error being so large because of the intrinsic uncertainty in defining the s quark mass, while from the measured values from [8]:

$$m_\tau = 1.777 \text{ GeV}, \quad m_\mu = 105.66 \text{ MeV}, \quad (8)$$

one obtains the estimate:

$$\theta_{\tau\mu} = 0.2463. \quad (9)$$

Assume now that the mass matrices of the U and D quarks as well as the charged leptons are all aligned at the same scale as proposed above, and plot the values of the rotation angles obtained before, all starting from the direction of the t quark state. One obtains then the Figure 3 where, in the planar approximation, we have taken $\theta_{ts} = \theta_{tb} + \theta_{bs}$, and $\theta_{t\mu} = \theta_{t\tau} + \theta_{\tau\mu}$, with θ_{tb} taken from (3) and $\theta_{t\tau}$ (indicated by a cross in Figure 3) estimated by interpolation between the values of θ_{tb} and θ_{tc} given above. One sees that all the gathered information can indeed be comfortably explained by a mass matrix rotating smoothly as the scale changes, as suggested.

The above analysis was done under the simplifying assumption of there being only 2 generations of fermion states, but we shall now show that it is already a very good approximation to the actual 3-generation situation. When 3 generations are introduced, the mixing matrix can be parametrized as:

$$\begin{pmatrix} V_{tb} & V_{ts} & V_{td} \\ V_{cb} & V_{cs} & V_{cd} \\ V_{ub} & V_{us} & V_{ud} \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}. \quad (10)$$

Notice that although this parametrization, which is more convenient for our purpose here, is formally the same as the original Kobayashi-Maskawa parametrization [9], the rows and columns are labelled differently, namely in order of decreasing mass rather than in order of increasing mass, so that the meanings of the angles are also different. But, as for the original Kobayashi-Maskawa parametrization, (10) can be interpreted as a product of 3 Euler rotations and a phase change [10]:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}, \quad (11)$$

where c_i and s_i are the cosines and sines of the Euler-like angles θ_i . One sees then that if we continue to denote as before V_{tb} as $\cos \theta_{tb}$, the elements V_{ts} and V_{cb} are no longer just given by $\sin \theta_{tb}$ but by respectively $\sin \theta_{tb} \cos \theta_3$ and $\sin \theta_{tb} \cos \theta_2$. However, the angles θ_2 and θ_3 , being still in the present picture just rotation angles undergone by the triad of state vectors from the scale m_t to m_b , are expected to be small, namely of the order of the difference in scale times the rotation rate. Indeed, their actual values can be estimated from

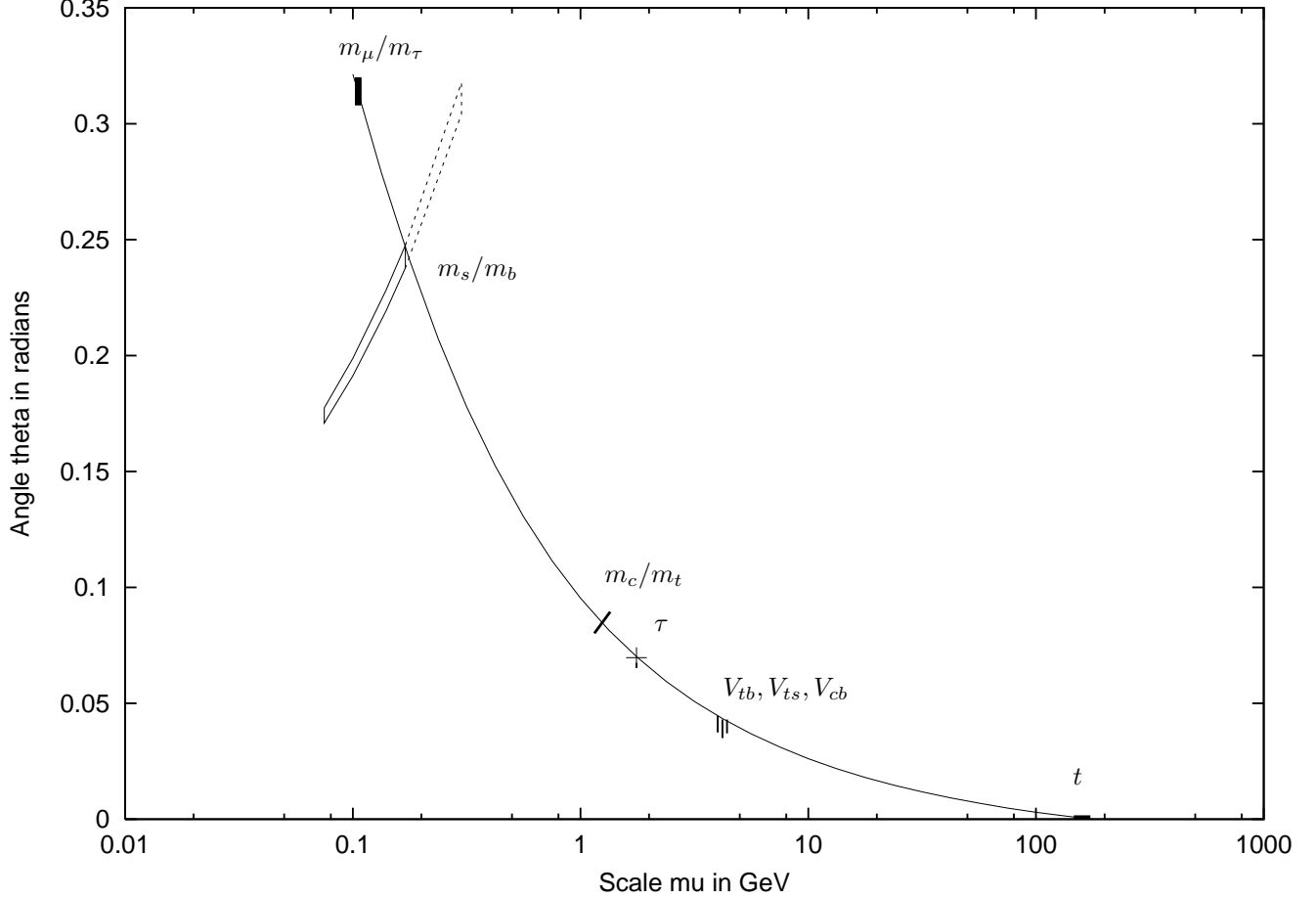


Figure 3: The rotation angle in the planar 2-generation approximation, as estimated from various mixing elements and mass ratios measured in experiment, is plotted as a function of the energy scale starting from the t mass. The error bars shown correspond to the errors in the empirical values of the mixing elements and masses quoted in [8], where in the error for the s quark (particularly large because of the intrinsic difficulty in defining the s mass) the solid bar represents the mass range defined at a scale of 2 GeV [8] while the dashed bar that defined at a scale of 1 GeV [11]. The curve shown is the result of an earlier calculation [12] with the DSM scheme as detailed in text.

the empirical values given in [8] for the corner elements of the CKM matrix in comparison to the values of V_{ts} and V_{cb} quoted above, giving:

$$\begin{aligned} |V_{td}| = 0.004 - 0.014 &\longrightarrow |\tan \theta_3| = 0.093 - 0.400, \\ |V_{ub}| = 0.002 - 0.005 &\longrightarrow |\tan \theta_2| = 0.047 - 0.135, \end{aligned} \quad (12)$$

from which one gets:

$$\cos \theta_2 = 0.999 - 0.991; \quad \cos \theta_3 = 0.996 - 0.928. \quad (13)$$

Hence, one concludes that in the 2 generation planar approximation of Figures 2 and 3 where one puts $V_{ts} = V_{cb} = \sin \theta_{tb}$, one has made an error of at most a few percent.

A similar error has been made in Figure 3 as regards the lower generation masses obtained from the “leakage” mechanism. The estimate (7) is for the angle rotated between the scales of m_b and m_s but in Figure 3 we have added this angle to the rotation angle from scale m_t to scale m_b to get the angle from scale m_t to scale m_s . Such an addition is valid in the 2 generation approximation but has nonplanar corrections in the actual 3 generation situation. In this case, there does not seem to be enough empirical information to evaluate the error directly, but its rough value can be inferred. The angle between the plane defined by the t and c vectors and the plane defined by b and s is given by the angle between their normals, namely the vectors for u and d respectively, which according to [8] takes the value:

$$|V_{ud}| = \cos \theta_{ud} = 0.9742 - 0.9757 \quad (14)$$

giving

$$\theta_{ud} = 0.2209 - 0.2276. \quad (15)$$

The nonplanar error incurred in the angle at scale m_s plotted in Figure 3 is of order $\cos \theta_{ud}$ and is thus of order a few percent, which is negligible given the large error already inherent in the definition of the s quark mass. A similar error is presumably present in the angle plotted in Figure 3 at scale m_μ , but one has at present no means for directly ascertaining this.

Thus, as far as the evidence goes in Figure 3, one seems justified in suggesting that both quark mixing and the second generation masses of quarks and charged leptons can arise from a continuous rotation with scale change of the fermion mass matrix. The analysis can in principle be extended further to lower scales to examine the masses of the u and d quarks, and eventually

even the electron and neutrino masses and lepton mixing, i.e. neutrino oscillations. For doing so, however, analyses with the full 3 generation rotation matrices involving 3 angles and a phase as detailed in (10) is necessary, and there is as yet insufficient empirical information to do so without relying on some extrapolation model.

For this reason, let us consider the curve shown in Figure 3. This appears to be the best-fit to the data points but is actually the result of a calculation [12] done two years ago with the Dualized Standard Model (DSM) scheme that we ourselves advocate. The DSM scheme first suggests a possible explanation for 3 fermion generations as the broken dual symmetry to colour, and then goes on to attempt a semi-quantitative understanding of the fermion mass and mixing patterns with reasonable success [13]. To achieve the latter purpose, it assumes that the fermion mass and mixing patterns both arise as consequences of the rotation of the mass matrix, exactly in the manner suggested by the above empirical analysis. The mass matrix rotation in this scheme is driven by a dual Higgs mechanism the full details of which can be found in e.g. [12, 13] but need not bother us at this juncture. We note here only the following two points of particular relevance to the present discussion. (a) The mass matrix has a high energy rotational fixed point at infinite scale with its heaviest eigenvector pointing in the direction $(1, 0, 0)$ and a low energy fixed point at zero scale with the heaviest eigenvector pointing in the direction $\frac{1}{\sqrt{3}}(1, 1, 1)$. (b) The rotation of the mass matrix between the two fixed points calculated to one-loop order depends on 3 parameters, namely 2 Higgs vev's and a Yukawa coupling, which were fitted to the data on fermion mass and mixing parameters, specifically in [12] to m_c/m_t , m_μ/m_τ , and the Cabibbo angle.

The fact (a) that there is a rotational fixed point at infinite scale means that the rotation will go slower at high energy, which is apparently what is indicated by the data in Figure 3. This is one reason why the DSM curve in Figure 3 is able to reproduce so well the scale dependence of the mass matrix rotation with only 2 parameters (the third having to do with the degree of nonplanarity as measured by the Cabibbo angle $\sim \theta_{ud}$). Conversely, the fact that the data points in Figure 3 by themselves already seem to follow a continuous rotation curve also goes some way towards explaining the at first sight somewhat puzzling numerical success of the DSM in fitting, seemingly so effortlessly, the empirical mass and mixing patterns.

That being the case, it seems worthwhile to explore even lower scales by attempting an extrapolation with the DSM formula. This has already been done for the lowest generation masses u, d and e [12]. The results fall into

a sensibly hierarchical pattern but are numerically inaccurate. Presumably, this means that the rotation curve calculated to one-loop order in the DSM, which has apparently worked over an already surprisingly large range of scales down to the μ mass because of angular proximity to the rotational fixed point at infinite scale, is no longer reliable further down. Nevertheless, it may still be worthwhile to consider with this extrapolation lepton mixing or neutrino oscillation, where even a qualitative answer would be instructive. Although in so doing an even further extrapolation is needed, there is, as we shall see, a saving grace by virtue of the low energy fixed point (a) at zero scale. Following then the previous precepts, one should presumably define the state vectors of neutrinos at their respective mass scales as for the other particles.² In particular, for the heaviest neutrino ν_3 , experiment suggests a mass of around 0.05 eV [3, 4], which is some 9 orders of magnitude below the last point at the scale m_μ shown in Figure 3. This means first that neutrino mixing angles can be considerably larger than those for quarks, which agrees with what has been observed experimentally, but it also means that without an accurate extrapolation formula, any estimate would seem at first sight quite hopeless.

Fortunately, with the DSM scheme, the extrapolation is saved by the fact that the mass matrix rotation has a fixed point at the scale $\mu = 0$, which means that the rotation is at least asymptotically bounded. Further, at $\mu = m_{\nu_3} \sim 0.05$ eV, one is likely to be already so near this low energy fixed point that one can safely approximate the state vector of the heaviest neutrino ν_3 just by the vector at the fixed point, namely: $\frac{1}{\sqrt{3}}(1, 1, 1)$. That this is indeed the case is confirmed by the calculation in e.g. [12]. Putting in then the state vectors obtained before in [12] for μ and e :

$$\begin{aligned} |\mu\rangle &= (-0.075925, 0.774100, 0.628494), \\ |e\rangle &= (0.027068, -0.628482, 0.777354), \end{aligned} \tag{16}$$

²In [14, 12], we had defined instead the state vectors of neutrinos at the scales of their respective Dirac masses, with which prescription we are thus now at variance. As we shall see, however, the result for the MNS elements are qualitatively similar, and are equally consistent with existing data, the reason being that the physical masses of neutrinos as well as their Dirac masses as found in [14, 12] are all already close to the fixed point at zero scale. From the DSM point of view, the present prescription means that the MNS matrix elements can now be predicted from the trajectory fitted with quark and charged lepton data without any further input from neutrinos; it also removes the requirement previously found necessary in [14, 12] of selecting only the vacuum solution to the solar neutrino problem. Details of these and other implications on the DSM will, we hope, be reported later in a separate communication.

one obtains immediately:

$$\begin{aligned} U_{\mu 3} &= \langle \mu | \nu_3 \rangle = 0.7660 \\ U_{e 3} &= \langle e | \nu_3 \rangle = 0.1016, \end{aligned} \tag{17}$$

where one notes that the vector $|e\rangle$, being defined as the vector normal to both $|\tau\rangle$ and $|\mu\rangle$, is already determined at the μ mass scale, and does not therefore suffer from the inaccuracy mentioned above in the determination of the e mass through extrapolation. These numbers, being near maximal for the “atmospheric” angle $U_{\mu 3}$ and small for the “Chooz” angle $U_{e 3}$, are well consistent with the present experimental limits of around $0.56 - 0.83$ and $0.00 - 0.15$ for $U_{\mu 3}$ [3, 4] and $U_{e 3}$ [15] respectively. We note that these predictions are quite robust. Although the actual numbers in (17) come from the state vectors (16) of μ and e , which depend on the details of the DSM calculation in [12], the conclusion that $U_{\mu 3}$ is large and $U_{e 3}$ small is a consequence only of the “leakage” mechanism illustrated by Figure 1 which dictates that the μ state vector should point in the direction of rotation. And so long as the rotation from the τ scale to the μ scale is in the general direction of the asymptotic rotation between the two fixed points at infinite and zero scales, the μ vector will lie roughly on the plane containing the two asymptotic vectors and the e vector be roughly normal to this plane, from which fact alone the qualitative conclusion that $U_{\mu 3}$ is large and $U_{e 3}$ small will already follow. However, if one goes one step further and approximates the state vector of the second heaviest neutrino by the tangent to the rotation trajectory at the fixed point, which now depends on how the fixed point is approached by the extrapolated rotation trajectory and therefore can be inaccurate, one obtains a rough value also for the “solar” angle $U_{e 2}$ of about 0.24, but this lies outside the present experimental range of around $0.4 - 0.7$.

Thus it seems that just with the general idea of mass matrix rotation, plus the two fixed points at infinite and zero scales as suggested by the DSM scheme but without the injection of its other details, one obtains already a consistent near-quantitative description of both quark and lepton mixing plus the fermion mass hierarchy, with most of the salient features (except for the moment CP-violation) included. The DSM comes in, apart from locating the fixed points, only in supplying a *raison d’être* for the 3 generations of fermions in the first place, and then an explicit theoretical mechanism for driving the mass matrix rotation, both of which are of course conceptually very important, but are not really essential for deriving the above numerical results.

An obvious next question is whether such a description can be independently tested, and the answer is that perhaps it can, which we consider one of its attractive features. Once the mass matrix rotates, then its eigenstates defined at one scale will in general no longer be eigenstates at some other scale. In particular, this means that the fermion flavour states defined each at its own mass scale as outlined above will not be diagonal states of the mass matrix at an arbitrary energy. Since reaction amplitudes depend in general on the fermion mass matrix, it follows that these too may become non-diagonal and lead, as has been suggested in [16], to new flavour-violating reactions differing in nature from those arising from, say, flavour-changing neutral currents. Such so-called “transmutation” effects have been studied in [16, 17] and in other cases and it is found that with a mass matrix rotating at the sort of speed indicated in Figure 3 they could be of sufficient size to be observable in some high sensitivity experiments in current operation such as Bepc, Cleo, BaBar, and Belle [18, 19, 20, 21]. However, although the observation of flavour-violation at the estimated magnitude favours its interpretation as coming from a rotating mass matrix as here advocated, the absence of the same is less conclusive since reaction amplitudes depend on quantities other than the fermion mass matrix, such as interaction vertices, which may also rotate giving rise to other effects modifying the above-cited estimates. Nevertheless, in view of the very significant implications on fermion properties discussed in this note, an experimental search for such possible transmutation effects due to a rotating mass matrix will seem to be an extremely worthwhile quest.

References

- [1] See e.g. A. Santamaria, G. Rodrigo, and M. Bilensky, hep-ph/9802359, talk given at the workshop Valencia97, published in the proceedings, *Beyond the Standard Model, from theory to experiment*, ed. I. Antoniadis, L.E. Ibanez, J.W.F. Valle, World Scientific, 1998.
- [2] See e.g. B. Grzadkowski, M. Lindner and S. Theisen, Phys. Lett. B198, 64, (1987).
- [3] Superkamiokande data, see e.g. talk by T. Toshito at ICHEP’00, Osaka (2000).
- [4] Soudan II data, see e.g. talk by G. Pearce, at ICHEP’00, Osaka (2000).

- [5] The idea of a possible “horizontal symmetry” linking generations is quite old. Examples of some early references are: F. Wilczek and A. Zee, Phys. Rev. Lett. 42, 421, (1979); A. Davidson and K.C. Wali, Phys. Rev. D20, 1195, (1979), D21, 787, (1980); T. Maehara and T. Tanagida, Prog. Theor. Phys. 61, 1434, (1979); T. Yanagida, Phys. Rev. D22, 1826, (1980).
- [6] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
- [7] See e.g. the article by C. Jarlskog in *CP Violation*, ed. C. Jarlskog, World Scientific, 1989.
- [8] Review of Particle Physics, D.E. Groom et al., Eur. Phys. Journ. C15 (2000) 1.
- [9] M. Kobayashi and T. Maskawa, Prog. Teor. Phys. 49, 652 (1973).
- [10] See e.g. J. F. Donoghue, E. Golowich, and B. R. Holstein, *Dynamics of the Standard Model*, Cambridge University Press, 1992, pp 60-69.
- [11] R. M. Barnett et. al, Phys. Rev. D54, 1 (1996).
- [12] José Bordes, Chan Hong-Mo and Tsou Sheung Tsun, hep-ph/9901440, Eur. Phys. J. C 10, 63, (1999).
- [13] See e.g. Chan Hong-Mo and Tsou Sheung Tsun, hep-ph/0008312 (2000), and/or Chan Hong-Mo hep-th/0007016, Int. J. Mod. Phys, A16, 163, (2001).
- [14] José Bordes, Chan Hong-Mo, Jakov Pfaudler, and Tsou Sheung Tsun, hep-ph/9802420, Phys. Rev. D58, 053003, (1998).
- [15] CHOOZ collaboration, M. Apollonio et al., Phys. Lett. B466, 415, (1999), hep-ex/9907037.
- [16] José Bordes, Chan Hong-Mo and Tsou Sheung Tsun, Phys. Rev. D63, 016006 (2001), hep-ph/0006338.
- [17] José Bordes, Chan Hong-Mo, Jacqueline Faridani, and Tsou Sheung Tsun, Phys. Rev. D63, 016009 (2001), hep-ph/0007004.
- [18] Bepc Collaboration, see e.g. Z.G. Zhao, talk given at APPAC 2000, hep-ex/0012056.

- [19] Cleo Collaboration, see e.g. Phys. Rev. D62, 051101, (2000).
- [20] BaBar collaboration, see e.g. D.G. Hitlin, talk given at the 30th International Conference on High energy Physics, Osaka, Japan, Jul.-Aug. 2000, hep-ex/0011024.
- [21] Belle Collaboration, see e.g. paper presented at the 7th International Conference on B Physics at Hadronic Machines, Kibutz Maagan, Israel, Sep. 2000, hep-ex/0101033.